Applications of Fourier series in communication system

Prof. Kalyani Hande, Prof. Farha Vanu.
Department of Enginering sciences and Humanities, Saraswati college of engineering Kharghar,
University of Mumbai
Kalyanihande21@gmail.com
farhavanu@gmail.com

Abstract: This work examine how to convert analog system to digital system by using Fourier series and its applications in communication system.

Key words: FT (Fourier transform).

INTRODUCTION

Mathematics is everywhere in every phenomenon, technology, observation, experiment etc. All we need to do is to understand the logic hidden behind. In this paper we are focusing on applications of Fourier series in communication system.
FT is named in the honour of Joseph Fourier (1768-1830), one of greatest names in the history of mathematics and physics. Mathematically speaking, The Fourier transform is a linear operator that maps a functional space to another functions space and decomposes a function into another Function of its frequency components.

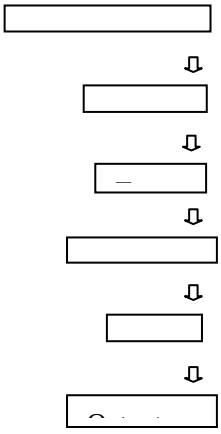
The Fourier series:
Fourier series = a finite sum of harmonically related sinusoids.
Mathematically,
The expression for a Fourier series is

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt$$

Where a_0, a_n, b_n are Fourier coefficients.

What is a communications system?

Systems designed to transmit and receive information.



Terminologies:

Input transducer: The device that converts a physical signal from source to an electrical, mechanical or electromagnetic signal more suitable for communicating.

Transmitter: The device that sends the transduced signal
Transmission channel: The physical medium on which the signal is carried
Receiver: The device that recovers the transmitted signal from the channel.
Output transducer: The device that converts the received signal back into a useful quantity.
(Ref. [2],[3],[8])

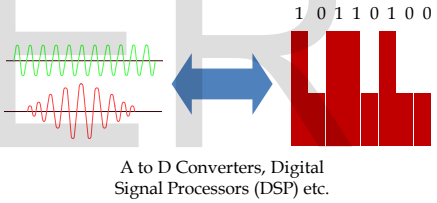


Fig 1: Analog to Digital conversion

Example of analog to digital conversion by using Fourier series:
Find the Fourier series of the following periodic function

$$f(t) = A \quad when \quad 0 < t < \pi$$
$$= -A \quad when \quad \pi < t < 2\pi$$

$$f(t + 2\pi) = f(t)$$

Solution: The expression for a Fourier Series is

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)dt$$
$$= \frac{1}{2\pi}\left[\int_0^{\pi} f(t)dt + \int_{\pi}^{2\pi} f(t)dt\right]$$
$$= \frac{1}{2\pi}\left[\int_0^{\pi} A\,dt + \int_{\pi}^{2\pi} -A\,dt\right]$$
$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt\,dt$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} A\cos nt\,dt + \int_{\pi}^{2\pi}(-A)\cos nt\,dt\right]$$
$$= \frac{1}{\pi}\left[A\frac{\sin nt}{n}\right]_0^{\pi} + \frac{1}{\pi}\left[-A\frac{\sin nt}{n}\right]_{\pi}^{2\pi} = 0$$

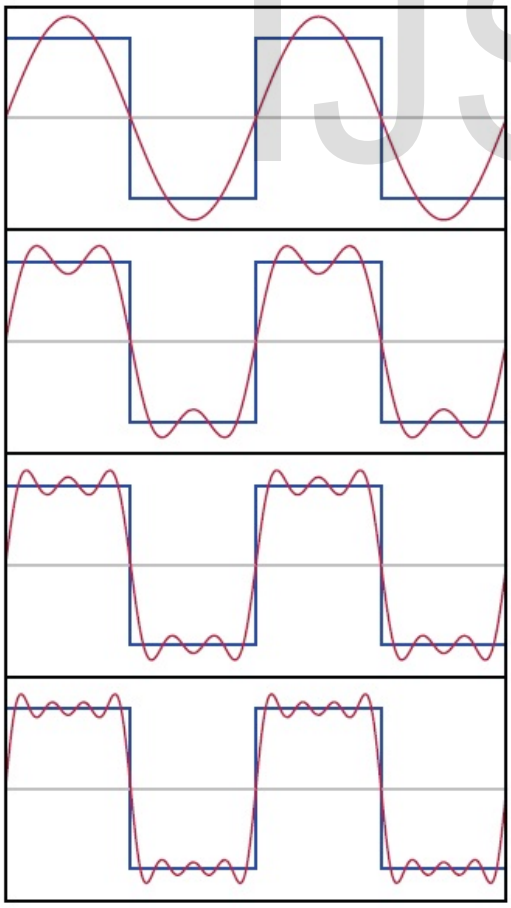
$$b_n = \frac{A}{n\pi}\left[-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi\right]$$
$$= \frac{A}{n\pi}\left[1+1+1+1\right]$$
$$= \frac{4A}{n\pi} \quad \text{when n is odd}$$

$$b_n = \frac{A}{n\pi}\left[-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi\right]$$
$$= \frac{A}{n\pi}\left[-1+1+1-1\right]$$
$$= 0 \quad \text{when n is even}$$

Therefore, the corresponding Fourier series is

$$f(t) = \frac{4A}{\pi}\left(\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \frac{1}{7}\sin 7t + \cdots\right)$$

(Ref.[1])



A square wave being approximated by a finite Fourier series with an increasing number of terms.

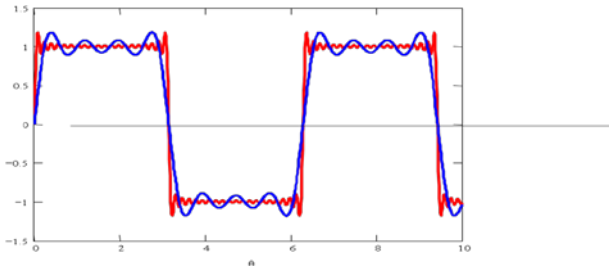


Figure 1: The red curve was drawn with 20 terms and the blue curve was drawn with 4 terms.

Thus the graph indicate that the series is convergent and has sum $f(x)$.

Review and conclusion:

Fourier transform is mathematical procedure which transforms function form time domain to frequency domain with communication signals.
And as we have seen Fourier converts signal from analog to digital, Fourier methods are commonly used for signal analysis and system design in modern telecommunications like cell phone networking also used in image processing systems, vibration analysis, optics, Qauntum machines.(Ref.[4])

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